SEE Rate Estimation:

Model Complexity and Data Requirements

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SEE Rate Estimates and Confidence

- [Ladbury, NSREC 2007}: Estimating confidence level of SEE rate
  - SEE numbers fluctuate about a mean, m, according to Poisson statistics

$$P(\mu, n) = \mu^n \exp(-\mu) / n! \qquad (1)$$

  - Expected # of SEE for LET_i

$$\mu_i = F_i \sigma_i = F_i \sigma_{lim}\left(1 - \exp\left(-\left(\left(LET_i - LET_0\right)/W\right)^s\right)\right) \qquad (2)$$

  - Use Likelihood to find best-fit and confidence contours of parameters: s_{lim}, LET_0, W, s

$$L = \prod_{i=1}^{n} P(\mu_i, n_i) \qquad (3)$$

  - Use Figure of Merit to indicate parameters likely to give high rates
  - Highest CREME96 rate for these 10 combinations gives WC rate at CL
- Unfortunately, CREME96 won't work for many State of the art parts
- Can we extend this technique beyond CREME96?

To be presented by Ray Ladbury at the 2008 Single Event Effects Symposium, Long Beach, CA, 16 April 2008

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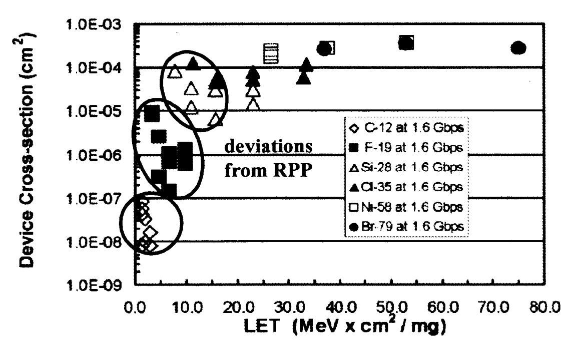
Outline

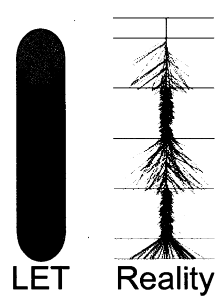
- Where does CREME96 break down?
- What are possible fixes?
- How do we decide on a new model?
  - Do we even need to decide on a single model?
- How do we compare results across models?

Breakdowns of CREME96 Model

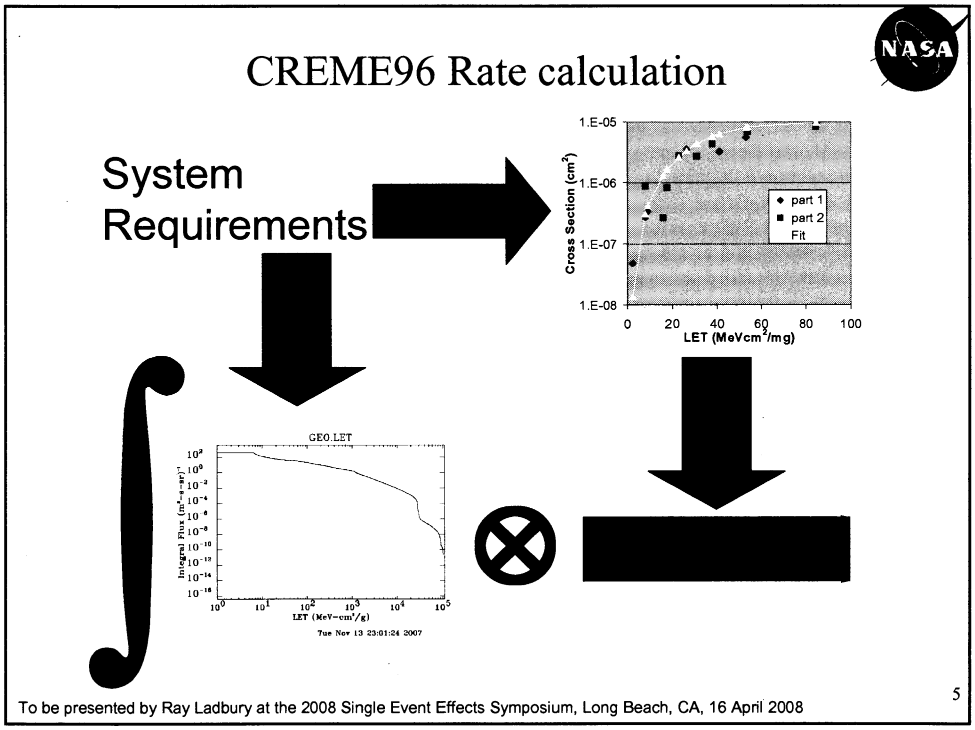
I.



II.



- Charge collection volume not RPP
- σ vs. LET curves may not follow a Weibull form (e.g. no saturation)
- Failure of LET concept and new charge generation mechanisms
  - May change expression for mean and/or add new error sources


CREME96 Rate calculation
System Requirements
1.E-05
1.E-06
1.E-07
1.E-08
Cross Section (cm²)
0 20 40 60 80 100
LET (MeVcm²/mg)
part 1
part 2
Fit
GEO.LET
Integral Flux (m²-s-sr)⁻¹
LET (MeV-cm²/g)
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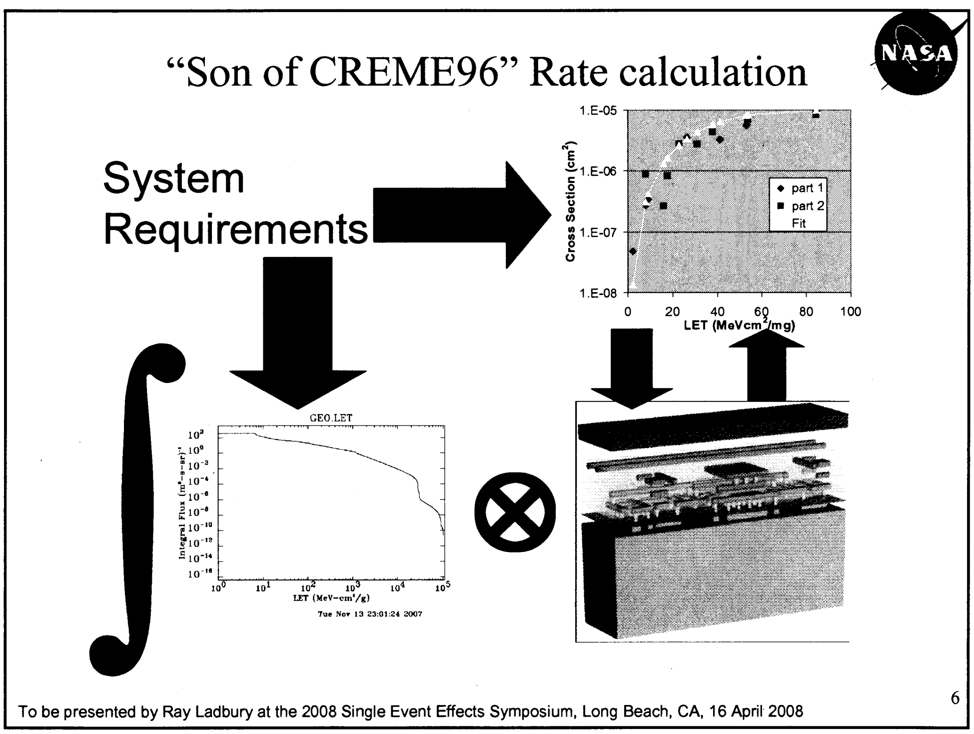
"Son of CREME96" Rate calculation
System Requirements
1.E-05
1.E-06
1.E-07
1.E-08
Cross Section (cm²)
0 20 40 60 80 100
LET (MeVcm²/mg)
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Rate Estimation Fidelity: A Series of Nested Models

REALITY

Monte Carlo models: anywhere from 0 to ∞ parameters

Improved physics from charge generation to circuit simulation
Detailed model of device and surroundings from data or fab.

CREME96: [illegible] parameters

Detailed environment model + transport
RPP Charge Collection Volume and simple departures from RPP (e.g. funnel)
Weibull (or LN) Form for σ vs. LET curve

Figure of Merit: 2-4 parameters
Approximate environment + geometry +charge generation and transport

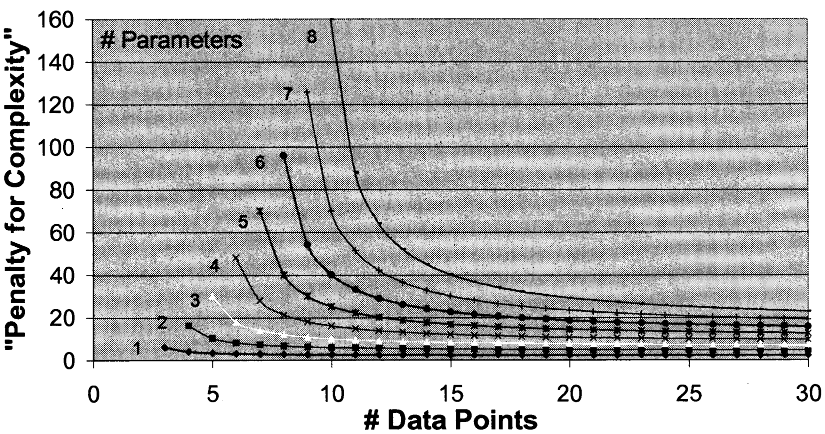
Choosing from many models

- Likelihood, L, can only compare modes w/ same complexity
  - More complicated models (> # parameters, k) give better fits
    - Example: Quadratic fit at least as good as a linear fit—even for linear data
- Information Theory approach
  - Hirotsugu Akaike's Information Criterion (AIC)
  - Asked: Given TRUE model, what is information lost as we move away?
  - Result: $$\mathrm{AIC} = 2(k - \ln L), \quad k = \#\text{ of parameters} \qquad (4)$$
  - For small datasets use corrected form

$$\mathrm{AIC_C} = 2(k - \ln L) + \frac{2k(k+1)}{(n-k-1)}, \; n = \#\text{of parameters} \qquad (5)$$

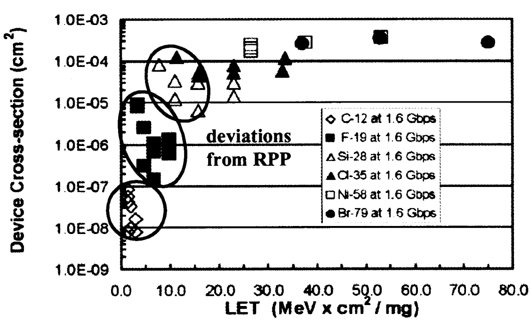
- Allows comparisons of models w/ different complexity
  - Model w/ smallest AIC→most predictive power (favors simpler models)
  - See Akaike, H., "A New Look at the Statistical Model Identification," IEEE Trans. Automatic Control **19** (6): 716-723 (1974)

What AIC Tells Us About Model Optimality

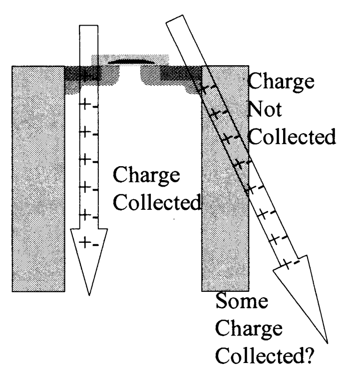


- AIC undefined for $n<k+2$
- Complex model must give MUCH BETTER fit than simple one
  - Penalty increases quadratically with # of parameters

Handling non-RPP Angular Dependence



- Angular dependence emerges naturally from physical model, but we may be insufficient without fitting to test data.

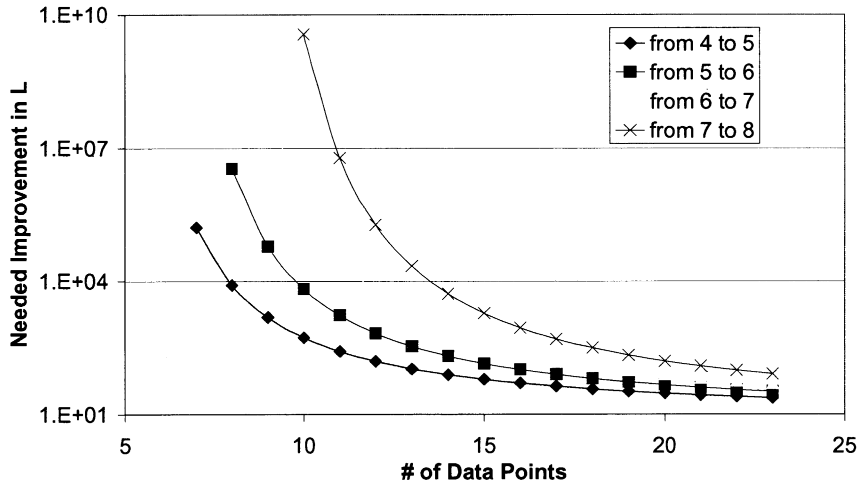


- $(LET,\theta) \sim \sigma_i(LET_i,\theta) = \sigma_{io}\cos^{m_i}(\theta)$
  - fit m_i to find m(LET)—or—
  - use $m_i \neq 0$ only when it is important
- Either way # model parameters, k, increases

$$P = 2k + \frac{2k(k+1)}{n-k-1}$$

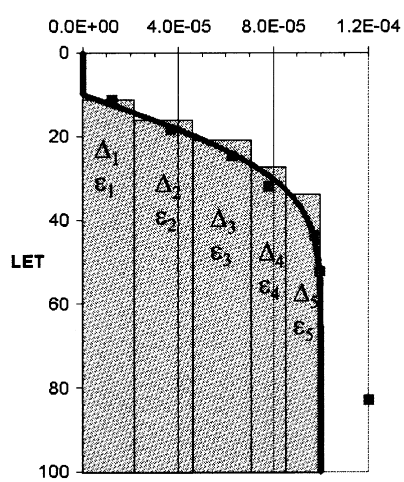
How much must Likelihood Improve?

Remember: AIC_c depends on ln(L)



Monte Carlo: Developing Models from Data

Cross Section



Without Manufacturer-supplied model:
Warren et al. [TNS 8/07, p. 898.]

- LET_0 determines Qcrit
- Increase in σ gives thickness of new sensitive vol.
- LET_i / LET_0 determines charge collection efficiency for ith sensitive vol., ε_i
- Fit σ vs. LET to Weibull

Similar to usual CREME approach

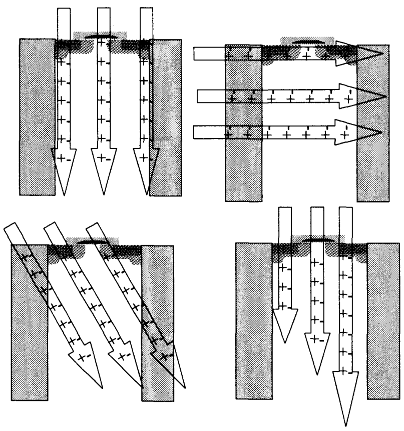
- Still subject to Poisson errors
- Amenable to likelihood/AIC treatment

except

- Cannot assume effective LET
- # of sensitive volumes may depend on when fit saturates and desired accuracy

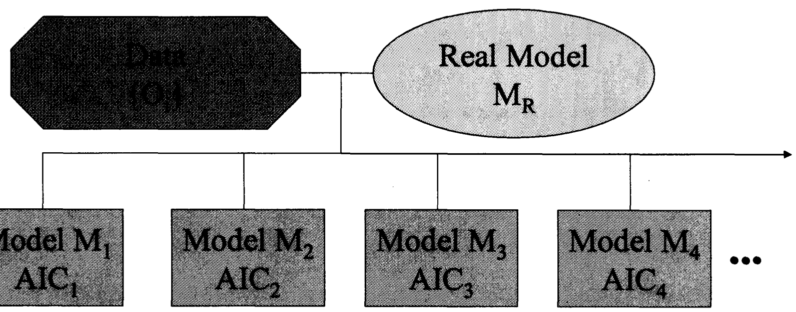
Verifying a Monte Carlo Rate

- Rate calculated done by Monte Carlo simulation w/ CAD model
- Verification: Look at model predictions vs. E, LET, θ...



- Monte Carlo predicts μ_i SEE counts for flux, F_i, at LET_i, E_i, incident @ angle θ_i. Observed=N_i
  - How Significant is disagreement?
    - Poisson(μ_i,N_i)
    - Other sources of error?
      - MBU, New error modes, etc.?
- Do we need to modify our model?
  - Do error trends tell us how?
    - Angle? Energy?
    - Frontside vs. Backside?
- Can also look at multiple models
  - over uncertainties over device parameters
  - AICc selects models that fit data most efficiently

Akaike Weights and Model Averaging



Introduce Akaike Weights: $W_j = \frac{\exp(0.5 * AIC_j)}{\sum_I \exp(0.5 * AIC_I)}$

Model Averaging: $\overline{R} = \sum_i W_i R_i$

W_j measures degree of support in $\{O_i\}$ for Model M_j

Bounding Rates Using AIC

S \ W	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
60	-88	-66	-49	-35	-24	-15	-9.8		
62	-71	-51	-36	-23	-14	-8.2			
64	-56	-39	-25	-15	-7.7				
66	-44	-29	-17	-8.5					-7.6
68	-34	-20	-10						-13
70	-26	-14						-11	-19
72	-19	-9					-9.4	-17	-27
74	-14					-7.7	-15	-24	-36
76	-9.8					-13	-22	-33	-46
78	-6.9				-10	-18	-29	-42	-57
80				-7.8	-15	-25	-38	-52	-68
82				-12	-21	-33	-47	-62	-80

- SEE rate @ confidence level CL defined by highest rate over all parameters with likelihood within δ_{CL} of best fit parameters.
- Use AIC to bound rate across models as well as across parameter values.

[redacted] [redacted] Model M_3 R_3, AIC_3 [redacted]

Ex: M_3=BF, M_2 w/in CL, M_1, M_4 outside CL→R_{WC}=MAX(R_2,R_3)

Conclusions

- Statistical Methods outlined in [Ladbury, TNS2007] can be generalized for Monte Carlo Rate Calculation Methods
- Two Monte Carlo Approaches
  - Rate based on vendor-supplied (or reverse-engineered) model
    - SEE testing and statistical analysis performed to validate model
  - Rate calculated based on model fit to SEE data
    - Statistical analysis very similar to case for CREME96
- Information Theory allows simultaneous considerationof multiple models with different complexities
  - Model with lowest AIC usually has greatest predictive power
  - Model averaging using AIC weights may give better performance if several models have similar good performance
  - Rates can be bounded for a given confidence level over multiple models, as well as over the parameter space of a model